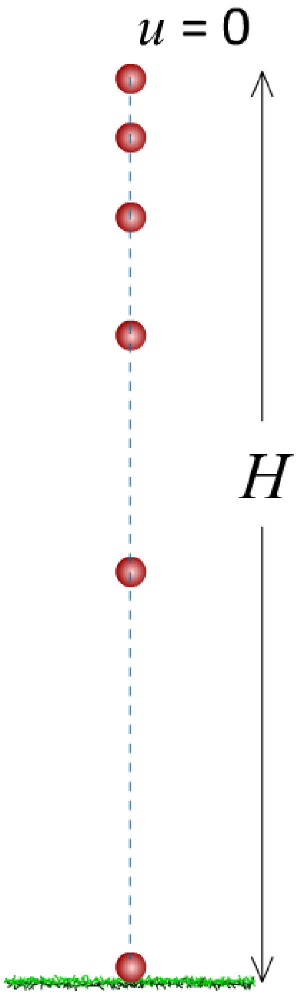
Freely falling body
&
Vertically projected body

Freely falling body

A body that is dropped and moves under the influence of only the gravitational force is called a freely falling body



Important points regarding a freely falling body

- Its initial velocity is zero ($u = 0$)
- Acceleration is due to gravity (9.8 ms^{-2} and downwards)
- The body gains velocity as it reaches the ground
- It is a non-uniform motion with constant acceleration
- Effect of air is assumed to be negligible

Quantities to analyze/determine

- Time of descent
- Final velocity of the body
- Average velocity
- Displacement in n^{th} second

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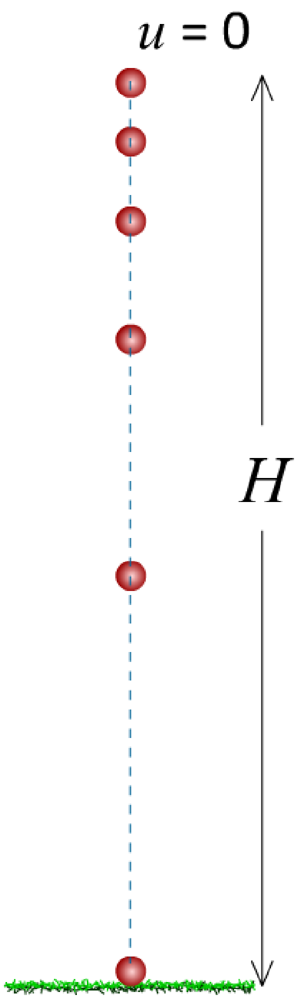
Consider a body dropped from a height H. Initial velocity (u) of the body is zero. As the body descends to ground, its velocity increases due to acceleration due to gravity (g).



Time of descent

It is the time taken by a freely falling body to reach the ground.

For a body moving with constant acceleration, displacement, as a function of time, is given by

$$S = ut + \frac{1}{2}at^2$$

Using u = 0 and a = - g (as it is downwards) and S = -H (also downwards) we get

$$-H = \frac{1}{2}(-g)t^2$$

$$\boxed{TOD = \sqrt{\frac{2H}{g}}}$$

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Final velocity

It is the velocity with which a freely falling body reaches the ground.

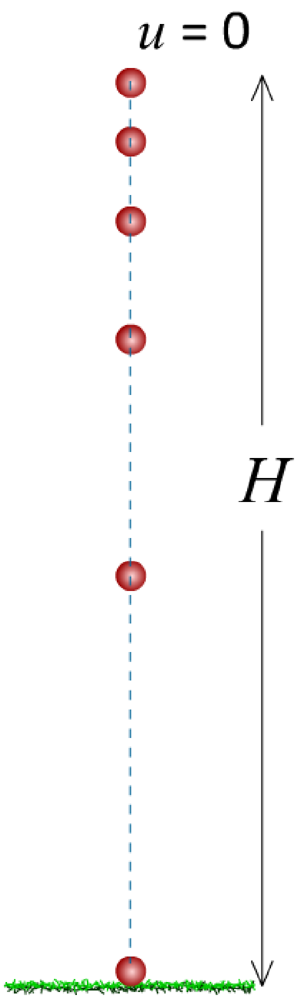
For a body moving with constant acceleration, velocity, as a function of displacement, is given by

$$v^2 - u^2 = 2aS$$

Using u = 0 and a = - g (as it is downwards) and S = -H (also downwards) we get

$$v^2 - 0 = 2(-g)(-H)$$

$$\boxed{v = \sqrt{2gH}}$$



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Average velocity

Average velocity is given by

$$v_{\text{avg}} = \frac{\Delta S}{\Delta t}$$

$$v_{\text{avg}} = \frac{0 - H}{TOD - 0}$$

Using the expression for *TOD* we get

$$v_{\text{avg}} = \frac{-H}{\sqrt{\frac{2H}{g}}}$$

$$v_{\text{avg}} = -\sqrt{\frac{Hg}{2}}$$

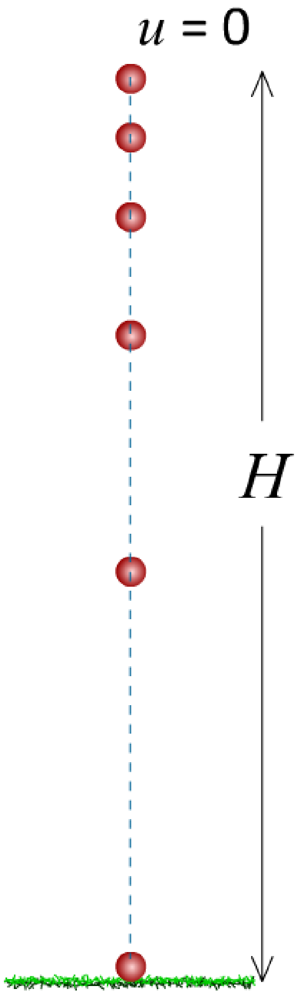
Alternate approach

Average velocity (*in case of constant acceleration only*) is also given by

$$v_{\text{avg}} = \frac{\bar{\boldsymbol{u}} + \bar{\boldsymbol{v}}}{2}$$

$$v_{\text{avg}} = \frac{0 + (-1)\sqrt{2gH}}{2}$$

$$v_{\text{avg}} = -\sqrt{\frac{Hg}{2}}$$



Ratio of displacements in n[th] second of descent

Displacement in the n[th] second is given by $S_n = u + a\left(n - \frac{1}{2}\right)$

Using u = 0 and a = - g (as it is downwards) we get $S_n = -g\left(n - \frac{1}{2}\right)$

n = 1 $S_1 = -g\left(1 - \frac{1}{2}\right) \Rightarrow -1\frac{g}{2}$

n = 2 $S_2 = -g\left(2 - \frac{1}{2}\right) \Rightarrow -3\frac{g}{2}$

n = 3 $S_3 = -g\left(3 - \frac{1}{2}\right) \Rightarrow -5\frac{g}{2}$

Therefore ratio is given by

$$S_1 : S_2 : S_3 = 1 : 3 : 5$$

Ratio of displacements in 1, 2, 3 seconds

Displacement of a body moving with uniform acceleration is given by

$$S = ut + \frac{1}{2}at^2$$

Using u = 0 and a = - g (as it is downwards) we get

$$S = -\frac{1}{2}gt^2$$

t = 1 $\quad S = -\frac{1}{2}g\,1^2 \Rightarrow S(1) = -\frac{1}{2}g$

t = 2 $\quad S = -\frac{1}{2}g\,2^2 \Rightarrow S(2) = -\frac{1}{2}4g$

t = 3 $\quad S = -\frac{1}{2}g\,3^2 \Rightarrow S(3) = -\frac{1}{2}9g$

$$S(1):S(2):S(3) = 1\frac{g}{2}:4\frac{g}{2}:9\frac{g}{2}$$

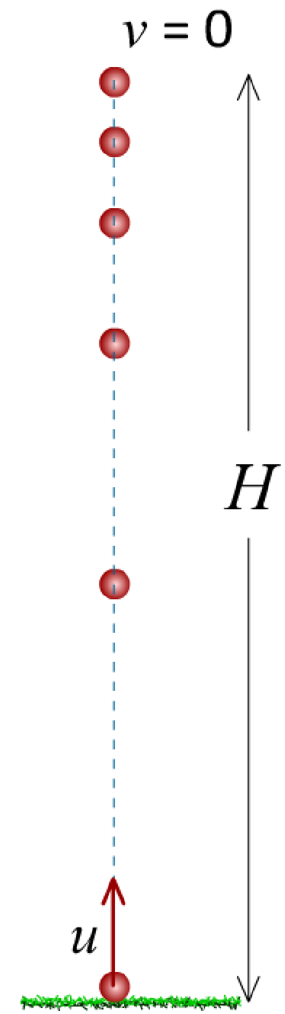
$$S(1):S(2):S(3) = 1:4:9$$

Vertically projected body

A body projected vertically up with an initial velocity executes 1-D motion under the influence of gravitational force.

Important points regarding a vertically projected body

- Its initial velocity is NOT zero
- Acceleration is due to gravity (9.8 ms^{-2} and downwards)
- The body loses velocity during ascent and gains velocity during descent
- It is a non-uniform motion with constant acceleration
- Effect of air is assumed to be negligible



Quantities to analyze/determine

- Time of ascent (TOA)
- Time of descent (TOD)
- Time of flight (TOF)
- The maximum height reached
- Additional observations

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Motion in a straight line

Consider a body projected vertically up with an initial velocity u. During ascent its velocity decreases, becomes zero momentarily at the highest point, and then increases during descent. This is due to acceleration due to gravity (g).

Time of ascent (TOA)

It is the time at which velocity of the body becomes zero momentarily

Or

It is the time taken by the body to reach the maximum height

Instantaneous velocity as a function of time, of a body moving with constant acceleration is given by

$$v = u + at$$

In this case a = - g (as it is downwards) and at the highest point v = 0 therefore

$$0 = u + (-g)t$$

$$\Rightarrow \quad t = \frac{u}{g}$$

$$\boxed{TOA = \frac{u}{g}}$$

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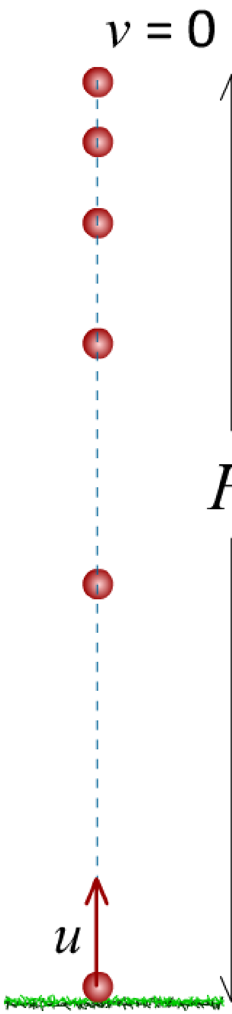


Maximum height (H)

It is the height at which the velocity of the body becomes zero momentarily

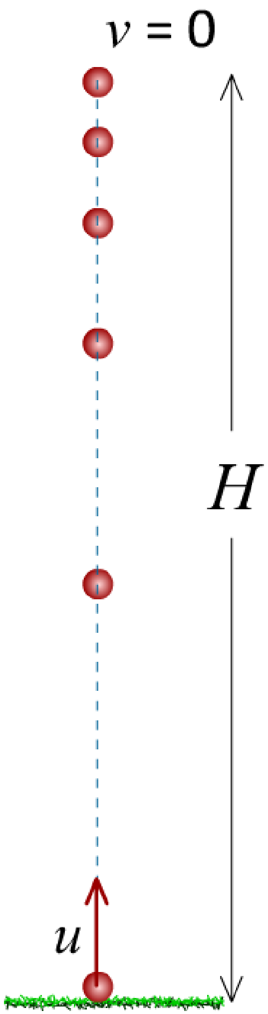
Instantaneous velocity as a function of displacement, of a body moving with constant acceleration is given by

$$v^2 - u^2 = 2aS$$

Using $v = 0$ and $a = -g$ (as it is downwards) and $S = H$ we get

$$0 - u^2 = 2(-g)(H)$$

$$\boxed{H = \frac{u^2}{2g}}$$



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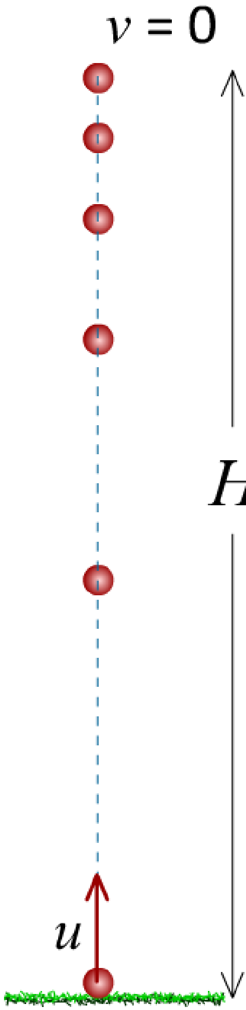
Time of descent (*TOD*)

It is the time taken by a vertically projected body to reach the ground , from the instant when its velocity is momentarily zero.



Considering the descent of the body from the highest point we get $u = 0$, $a = -g$ and $S = -H$

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad -H = 0 + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \quad H = \frac{1}{2}gt^2$$

Using the expression for maximum height $\dfrac{u^2}{2g}$ we get

$$\frac{u^2}{2g} = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t^2 = \frac{u^2}{g^2}$$

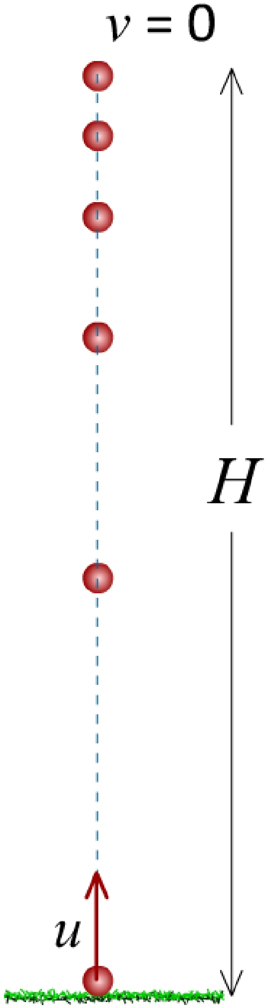
$$\Rightarrow \quad t = \frac{u}{g}$$

$$\boxed{TOD = \frac{u}{g}}$$

Comparing this with TOA it is observed that $TOA = TOD$.

Time of flight (TOF)

It is the time taken by a vertically projected body to reach the ground , from the instant it is projected up.



Since TOA is equal to TOD

$$TOF = 2 ? TOA$$

$$TOF = 2 ? \frac{u}{g}$$

$$\boxed{TOF = \frac{2u}{g}}$$

Additional observations

- Total displacement of the body is zero
- Total distance covered by the body is $2H$
- Average velocity for the complete trip is zero
- Average speed for the complete trip is $u/2$
- Ascent and descent are symmetric
- Distance covered by the body in 1^{st} second of its ascent is equal to the distance covered by it in the last second of its descent
- Distance covered by the body in the last second of its ascent is equal to the distance covered by it in the 1^{st} second of its descent

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